IDENTIFICATION, ESTIMATION AND CONTROL OF UNCERTAIN DYNAMIC SYSTEMS: A NONPARAMETRIC APPROACH

Nadine Hilgert[1], Vivien Rossi[2], Jean-Pierre Vila[1] and Vérène Wagner[3]

[1] UMR Analyse des Systèmes et Biométrie, ENSA.M - INRA, 2 place Viala, Bât. 29, 34060 Montpellier Cedex 1, France. hilgert@ensam.inra.fr, vila@ensam.inra.fr

[2] I3M, UMR CNRS 5149, Université Montpellier 2, cc51, Place Eugène Bataillon, 34095 Montpellier Cedex 5, France. Vivien.Rossi@ensam.inra.fr

[3] Institut de Veille Sanitaire, Département Santé et Environnement, 12 rue du val d'Osne 94415 Saint-Maurice Cedex, France.

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ABSTRACT

This paper is devoted to a presentation of the authors' practise of the nonparametric estimation theory for the estimation, filtering and control of uncertain dynamic systems. The fundamental advantage of this approach is a weak dependency on prior modeling assumptions about uncertain dynamic components. This approach appears to be of great interest for the control of general discrete-time processes, and in particular biotechnological processes, which are emblematic of nonlinear uncertain and partially observed systems.

1. INTRODUCTION

This paper is devoted to a survey of consistent applications of the nonparametric estimation theory for estimating, filtering and control of uncertain dynamic systems. It relies on a set of works the authors have been developing for more than ten years which emphasize the efficiency of these nonparametric tools in functional estimation as well as in probability density estimation in the context of controlled dynamic systems.

Therefore this presentation does not pretend to give the state-of-art in the field and an exhaustive survey of the applications of the nonparametric estimation theory to dynamic systems is out of the scope of the paper. The interested reader will take advantage to

consult recent works, as for example (Fan & Yao, 2003) for a general approach of the issue and (Greblicki, 1997), (Greblicki, 2002), (Greblicki, 2004) for a more specialized control engineering point of view, in addition to that we used and referenced in the paper.

The frame of the approach is that of the control of general discrete-time processes, and in particular biotechnological processes, which are emblematic of nonlinear uncertain and partially observed systems. The field of bioprocess modeling and control offers typical examples of structural time-variations problems which cannot be handled by classic control methods: the dependence of the kinetic coefficients on biomass and substrate state variables is affected by functional fluctuations and not merely parametric ones. In that case, a more appropriate approach would be robust control, in which uncertainty is explicitly accounted for at the beginning of the control design through numerical or functional bounds. However, the performance of the related controllers can be sensitive to settings that are too much conservative or too much optimistic. The nonparametric approach is free from these prior assumptions: through a stochastic learning process, uncertain functional components are progressively and automatically estimated as deterministic or random functions of the measured quantities, in accordance with their actual but unknown and possibly time-varying structures. The use of this functional estimation procedure, compared with the usual and more or less arbitrary choice of these model components, contributes to the reduction of one source of model inadequacy. Moreover, the stochastic frame in which these nonparametric models are designed allows some uncontrolled disturbances such as measurement errors and parameter variations to be accounted for.

In the following we shall present successively application of this nonparametric approach to identification, filtering and control of dynamic systems.

2. FRAMEWORK

The uncertain processes under consideration belong to the general class of controlled Markov chains. They are represented by discrete-time autoregressive models of the following type:

$$X_{t+1} = F_t(X_t, U_t, \varepsilon_{t+1}), \tag{1}$$

where $X_t \in \mathbb{R}^s$, $U_t \in \mathbb{R}^m$ and ε_t are the output, input and noise of the system, respectively. The driving function F_t may be completely or partially unknown, according to the degree of uncertainty in the analytical knowledge of the process. This function may be deterministic or stochastic and is supposed to obey some regularity conditions (see §2.2.). Moreover, when the state variable X_t is not observed, an observation model is supposed to be available:

$$Y_t = G_t(X_t, U_t, \eta_t) \tag{2}$$

where $Y_t \in \mathbb{R}^q$ and G_t is a known function and η_t an observation noise.

Estimating function F_t in model (1) may be intricate. The following particular case with an additive noise is more frequently met in practice:

$$X_{t+1} = f_t(X_t, U_t) + \varepsilon_{t+1}, \tag{3}$$

in which function f_t, from $\mathbb{R}^s \times \mathbb{R}^m$ to $\mathbb{R}^s$, may be completely or partially unknown. We are more particularly interested in a type of non-linear models where the control variable U_n acts in a known part of function f_t, such as models in the field of bioprocess modeling and control. They are of the form:

$$X_{t+1} = A_t(X_t)g_t(X_t) + B_t(X_t, U_t) + \varepsilon_{t+1}, \tag{4}$$

where A_t and B_t are known functions and function g_t is unknown. g_t can represent the growth rate of some microorganism population whose concentration is a component of the state variable X_t. The control variable U_t is for example the dilution rate of a polluted water at the entrance of a bioreactor.

Other examples of model (3) are for instance the evolution models of bacteria populations in food under the influence of environment covariates (U_t), or, in another field, models that describe the position of a space craft under control.

2.1. DEFINITIONS

We define a control policy, or strategy, as a sequence of deterministic mappings $d = (d_t)$, $t \geq 0$, from $(\mathbb{R}^s)^t$ to the space of controls $\mathcal{U}$, such that $U_t = d_t(X_1, \ldots, X_t)$. For all $x \in \mathbb{R}^s$

we shall consider the set of admissible controls with respect to x, to be a subset $A(x)$ of $\mathcal{U}$, for which $d_t(x_1, \ldots, x_{t-1}, x) \in A(x)$. A policy (d_t) will be said to be *A-admissible*, or *admissible* for short, if $\forall t,\ U_t \in A(X_t)$.

Moreover, Model (1) is said to be *stabilizable* by the use of any admissible policy, if, for any $\xi > 0$, there exists a compact set $\mathcal{C}$ of $\mathbb{R}^s$ satisfying the following property: for any initial law of X_0 and any admissible strategy d,

$$\liminf_{t\to\infty} \frac{1}{t+1} \sum_{i=0}^{t} \mathbb{1}_{\mathcal{C}}(X_i) \geq 1 - \xi \quad a.s., \tag{5}$$

where $\mathbb{1}_{\mathcal{C}}$ stands for the indicator function of the set $\mathcal{C}$. Sufficient conditions for this last property are introduced in the next paragraph.

2.2. ASSUMPTIONS

The following assumptions underlie most of the classic works in functional estimation of controlled nonlinear Markovian systems (Duflo, 1997). In the framework of bioprocesses they are not always easy to check, in particular assumptions 1 and 3. The satisfaction of these assumptions depends on the prior knowledge about the system.

The following set of assumptions about the noise ε will be needed.

Assumption 1 *The noise $\varepsilon = (\varepsilon_n)$ is a sequence of independent and identically distributed (i.i.d. for short) random vectors with mean 0 and covariance matrix Γ. Its distribution probability function is absolutely continuous (with respect to the Lebesgue measure), with a probability density function p supposed to be positive and C^1-class, and p and its gradient are bounded. ε admits a finite moment of order m_ε strictly greater than 2.*

Let us note that Assumption 1 is satisfied with a Gaussian white noise.

Assumption 2 *There exists a constant $w < 1$ such that*

$$\limsup_{||x||\to\infty} \frac{\sup_{i\in\mathbb{N}} \sup_{u\in A(x)}(||f_i(x,u)||)}{||x||} \leq w \quad a.s.$$

This last condition implies the stabilizability of the systems when Assumption 1 holds, for the particular cases of models (3) and (4), where the noise appears additively.

Assumption 3 *There exists a finite constant R such that, for all initial law of X_0 and all admissible control policy,* $\liminf_{t\to\infty} \frac{1}{t} \sum_{i=0}^{t-1} \mathbb{1}_{\|f_i(X_i,U_i)\|\leq R} > 0 \quad a.s.$

This assumption is weaker than the stabilizability condition (5). It is in fact a consequence of the stabilizability condition when Assumption 1 holds.

The global behaviour of the unknown set of stochastic functions f_t (resp. g_t) must be quite "stable". The following conditions imply this requirement:

Assumption 4 *The sequence (f_t) (resp. (g_t)) is a.s. equicontinuous and verifies one of these conditions*
(a) f_t (resp. (g_t)) converges a.s. uniformly on x and u to an unknown function f (resp. g)
(b) (f_t) (resp. (g_t)) is an i.i.d. sequence of mean f (resp. g), an unknown function of finite norm.

Note: Assumption 4 holds if (f_t)(resp. (g_t)) is a constant or continuous function f(resp. g).

Finally, in the special class (4) of bioprocess models, we need an additional assumption, on matrix A_t. Let A_t^- denote a general inverse, assumed to verify the following:

Assumption 5 $\forall r > 0,\ \sup\{||A_t^-(x)||\,;\, t \geq 0, ||x|| \leq r\} < \infty.$

2.3. APPLICATION: A BIOPROCESS MODEL

Let us consider the basic dynamics of a microbial growth in a stirred tank reactor, which in the case of one population of microorganisms on a single limiting substrate, is most often described by the following system (see for example (Bastin & Dochain, 1990))

$$\begin{cases} B_{t+1} = (1 + T(\mu_t - U_t))B_t + \varepsilon_{t+1}^1 \\ S_{t+1} = S_t - T\mu_t B_t/\tau + U_t(S_{in} - S_t)T + \varepsilon_{t+1}^2 \end{cases} \tag{6}$$

where the state variables B_t and S_t are the biomass (microorganisms) and substrate concentrations respectively, U_t the dilution rate is the control variable, S_{in} is the substrate

concentration in the influent, τ is the yield coefficient of the substrate consumption by the biomass, T is the sampling period. S_{in}, τ and T are known constants. The parameter of interest here is μ_t, the microbial growth rate, which is an uncertain time-varying function of the state. $\varepsilon = {}^t(\varepsilon^1, \varepsilon^2)$ is a white noise. This model is used to describe batch ($U_t = 0$) as well as fed-batch or continuous ($U_t \neq 0$) operating conditions.

The growth rate μ_t can be influenced by many factors: biomass concentration, substrate concentration, temperature, pH, ... For a given bioreaction this kinetic parameter is generally not well known, in spite of its crucial importance for a good modelling of the reaction dynamic. More than fifty models have been proposed for μ in the literature (see (Bastin & Dochain, 1990)). This model uncertainty is worse than unsatisfying and then a nonparametric approach could be appropriate to identify μ_t. It is easy to see that (6) enters the special class of models (4), and also (3).

3. IDENTIFICATION AND ESTIMATION OF NONLINEAR STOCHASTIC PROCESSES

In this section two quite different contributions of the nonparametric estimation theory to the study of the nonlinear Markovian processes described previously, are proposed.

The following subsection is devoted to the identification of model (3) and model (4) when these models are unknown or partially unknown, with state X_t completely observed. The convolution kernel method is applied to estimate function f_t (or only a subpart of it).

In subsection 3.2. the state variables X_t will not be supposed to be observed anymore and the issue considered will be that of their estimation (filtering) from knowledge of the observed variables Y_t and assuming knowledge of model F_t in (1) and model G_t in (2).

3.1. MODEL IDENTIFICATION WITH CONVOLUTION KERNEL ESTIMATORS

Kernel smoothing methods are among the most renowned nonparametric estimation and prediction methods. They belong to the family of smoothing methods (orthogonal polynomials, splines,...) and are based on a local averaging procedure. They are widely used to estimate probability density functions and regression functions, see (Bosq, 1996).

When the whole function f_t is unknown in model (3), the following semi-recursive kernel estimator, derived from that of Nadaraya-Watson (non recursive) (Bosq, 1996), can be

advantageously considered from the point of view of on-line computing:

$$\forall x \in \mathbb{R}^s \text{ and } u \in \mathbb{R}^m \quad \widehat{f}_t(x,u) = \frac{\sum_{i=0}^{t-1} \delta_{1,i}^{-s} \delta_{2,i}^{-m} K_1(\frac{x-X_i}{\delta_{1,i}}) K_2(\frac{u-U_i}{\delta_{2,i}}) X_{i+1}}{\sum_{i=0}^{t-1} \delta_{1,i}^{-s} \delta_{2,i}^{-m} K_1(\frac{x-X_i}{\delta_{1,i}}) K_2(\frac{u-U_i}{\delta_{2,i}})}, \tag{7}$$

The functions K_1 and K_2 are two kernel functions: they are real positive symmetric functions integrating to one.

The sequences $(\delta_{1,i})$ and $(\delta_{2,i})$, called the bandwidths, have to be positive and decreasing. See (Georgiev, 1984) for the case of an *i.i.d.* sequence (U_t), and (Wagner & Vila, 2001) for a more general situation.

In the case of biotechnological processes, the partially known model (4) is the most frequently met. In that case, the kernel estimation of g_t is given by:

$$\widehat{g}_t(x) = \frac{\sum_{i=0}^{t-1} \delta_i^{-s} K(\frac{x-X_i}{\delta_i}) A_i^-(X_i)(X_{i+1} - B_i(X_i,U_i))}{\sum_{i=0}^{t-1} \delta_i^{-s} K(\frac{x-X_i}{\delta_i})} \quad \forall\, x \in \mathbb{R}^s. \tag{8}$$

A_i^- is a general inverse of matrix A_i and K is a kernel function and (δ_i) its bandwidth.

To simplify the presentation, let us first introduce the convergence results for the last estimator (8). To that aim, we require the following set of assumptions:

Assumption 6 *The common bandwidth $\delta_i := \rho i^{-\alpha}$, $i \in \mathbb{N}$, ρ a positive constant, is chosen, with $\alpha \in (0, 1/2s)$ (Duflo, 1997) and the kernel function K is supposed to verify one of the two following assumptions:*

(a) The kernel K has a compact support and is Lipschitz continuous.

(b) K is positive, bounded, Lipschitz continuous, such that $\int ||y|| K(y) dy < \infty$ and for $y \neq 0$, $K(y) = O\left(||y||^{-\beta}\right)$ where $\beta > \frac{\alpha s+1}{\alpha}$.

Assumption 6(a) is for example satisfied with the Epanechnikov kernel, and Assumption 6(b) with the Gaussian kernel, see (Härdle, 1990). Concerning the bandwidth parameters, the form $\delta_i = \rho i^{-\alpha}$ is one for which convergence results have been established (Duflo, 1997), (Portier & Oulidi, 2000), (Hilgert, Senoussi, & Vila, 2000). In some cases, an optimal choice of the bandwidth parameters can be determined by cross validation procedures, see (Vieu, 1991) for instance.

Theorem 1 (Hilgert, 1997) *Suppose that Assumptions 1, 4 and 5 hold. Then,*
(a) under Assumptions 2 and 6(a), for any $0 < \alpha < 1/2s$*, any admissible control policy and any initial probability distribution* ν *of* X_0*,* $\widehat{g}_n$ *converges a.s. to* g *uniformly on compact sets:*

$$\lim_{t\to\infty} \sup_{x\in C} \|\widehat{g}_t(x) - g(x)\| = 0 \quad a.s.$$

(b) let (v_t) *be a sequence of positive real numbers such that* $v_t = O(t^w)$*,* $w > 0$*. Under Assumptions 3 and 6(b), if* ε *is Gaussian, for any* $0 < \alpha < 1/2s$*, for any admissible control policy and any initial probability distribution* ν *of* X_0*,* $\widehat{g}_n$ *converges a.s. to* g *uniformly over dilated compact sets:*

$$\lim_{t\to\infty} \sup_{||x||\leq v_t} \|\widehat{g}_t(x) - g(x)\| = 0 \quad a.s.$$

Moreover these convergence results are extended to (7) in the case of model (3) when the control law U_t excites the system sufficiently: it is supposed to be of the form $U_t = h_t(X_t) + \gamma_t\zeta_t$, where ζ_t is a Gaussian noise, γ_t is a positive sequence decreasing to 0 and (h_t) is a uniformly bounded sequence of functions. With this general setting, the results of Theorem 1 still hold, see (Wagner, 2001).

Application

In the following, we supposed that the state ${}^t(B_t, S_t)$ has been observed at any instant and we considered the simple case where μ_t is an unknown time-varying function of the substrate concentration S, $\mu_t(S) = g_t(S)$. From a sequence $(S_0, S_1, \ldots, S_t)$ of observed substrate concentrations we defined, following (8), the kernel estimator of μ_t:

$$\widehat{\mu}_t = \widehat{g}_t(S_t) = \frac{\sum_{i=1}^{t-1} \frac{1}{\delta_i} K(\frac{S_t - S_i}{\delta_i})(S_{i+1} - S_i - TU_i(S_{in} - S_i))(\frac{-\tau}{TB_i})}{\sum_{i=1}^{t-1} \frac{1}{\delta_i} K(\frac{S_t - S_i}{\delta_i})} \tag{9}$$

The following case of a convergent sequence of unknown functions $g_t(.)$ is considered for simulations:

$$\mu_t = g_t(S_t) = (1 - a_t)\mu_t^{Monod} + a_t\mu_t^{Tessier}, \tag{10}$$

$$\text{where} \qquad \mu_t^{Monod} = \mu_{max}\frac{S_t}{\theta + S_t} \quad \text{and} \quad \mu_t^{Tessier} = \mu_{max}(1 - \exp(\frac{-S_t}{\theta})) \tag{11}$$

are the well known Monod and Tessier models for the growth rate $\mu(S)$ (see (Bastin & Dochain, 1990)). μ_{max} is the maximum growth rate and θ is the Michaelis-Menten constant. We took $a_t = \exp(-(t-1)^2/2\sigma)$, which yields that $\mu_1 = \mu_1^{Tessier}$ and $(\mu_t - \mu_t^{Monod}) \stackrel{t\to\infty}{\longrightarrow} 0$.

The sequence of functions (g_t) was then the convergent deterministic sequence given by

$$g_t(S) = (1 - a_t)\mu_{max}\frac{S}{\theta + S} + a_t\mu_{max}(1 - \exp(\frac{-S}{\theta})). \tag{12}$$

Under these specifications the system (6) checks all the assumptions required to prove that $\widehat{g}_t(S)$ is a strong consistent estimator of $g_t(S)$ for all S in a given compact set. Moreover it can be shown that $\widehat{\mu}_t$ is also a consistent estimator of μ_t under further appropriate conditions.

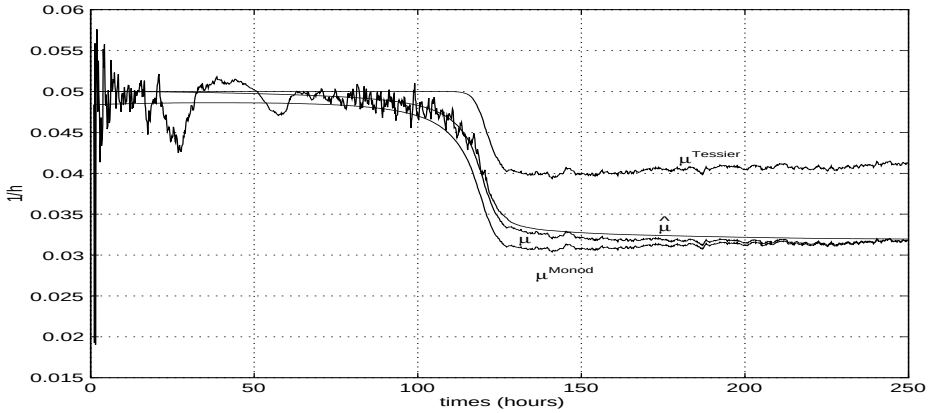

Figure 1: Trajectory of the simulated growth rate μ, obtained from the Monod and Tessier laws, and representation of the estimation $\widehat{\mu}$.

The behaviour of the kernel estimator $\widehat{\mu}_t$ is displayed in Figure 1, with the true μ_t trajectory simulated from (10) and with the original μ_t^{Monod} and $\mu_t^{Tessier}$ trajectories. The corresponding simulation of the process is given in Figure 2.

The computation have been done with the Epanechnikov kernel, the bandwidth $\delta_i = \rho i^{-\alpha}$ with $\rho = 4$ and $\alpha = 0.4$. Monod and Tessier models have been computed with $\theta = 1mg.l^{-1}$ and $\mu_{max} = 0.05h^{-1}$ and model (10) with $\sigma = 2 \times 10^5$. Model (6) has been simulated with $\text{var}(\varepsilon_t^1) = 10^{-2}$, $\text{var}(\varepsilon_t^2) = 5 \times 10^{-4}$, $B_0 = 1.2\, mg.l^{-1}$, $S_0 = 30\, mg.l^{-1}$, $\mu_0 = 0.05h^{-1}$, $T = 0.17\, h$, $S_{in} = 50\, mg.l^{-1}$, $\tau = 1$ and the control law $U_t = 1/(S_t + 30)$.

The convergence of $\widehat{\mu}_t$ towards μ_t was as expected and quite rapid.

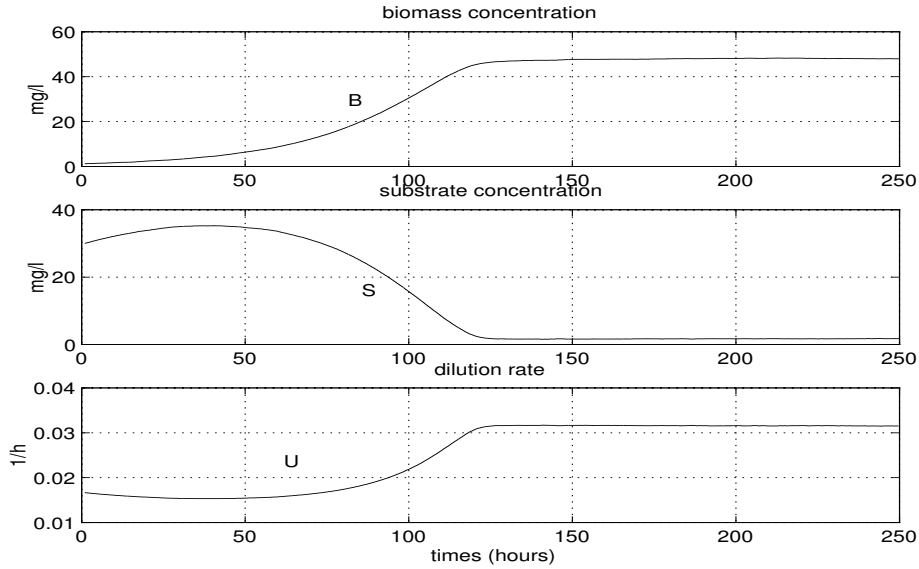

Figure 2: Simulation of the biological process (6).

3.2. ESTIMATION OF STATE VARIABLES WITH CONVOLUTION PARTICLE FILTERS

Besides its efficiency in functional estimation of uncertain models, as seen in the previous section, the nonparametric approach has proved to be useful as well in probability density estimation of unobserved state variables, *i.e.* in filtering problems.

The frame of this subsection, quite different from that of the previous one, is that given by model (1) and model (2), in which the functions F_t and G_t are now supposed to be completely known. On the other hand the state variables X_t are not observed anymore. The issue turns out to be the estimation of X_t or more generally that of the probability density function of X_t, from the analytical knowledge of the state model F_t (1), the observation model G_t (2) and the observed variables $Y_{1:t} = (Y_1, \cdots, Y_t)$. When F_t and G_t correspond to linear functions of X_t and U_t with additive noises, the well-known Kalman filter provides an optimal estimate of the probability distribution of X_t conditionally to $Y_{1:t}$, $P(X_t|Y_{1:t})$. In the other cases, only the so-called Monte Carlo filters or particle filters (see (Doucet, De Freitas, & Gordon, 2001) or (Del Moral, 2004)) provide consistent estimates of $P(X_t|Y_{1:t})$. The main principle of these filters is to build an estimate of $P(X_t|Y_{1:t})$ through the simulation of a

large number N of random state particles $\{x_i\}$ which are then weighted according to their likelihoods with respect to the observed variables up to time t.

However the usual particle filters require, in practice, the function G_t to be additive in the observation noise η_t, and the analytic form of the density of η_t to be known.

This last assumption really reduces the applicative potential of these particle filters. The convolution particle filters proposed in (Rossi, 2004) and (Rossi & Vila, 2004) drop this assumption thanks to the use of convolution kernels to estimate the conditional density $p(X_t|Y_{1:t})$, supposed to exist. The following algorithm shows the implementation of the Resampled-Convolution Filter (R-CF), one of the filters we developed (Rossi, 2004):

Starting from a given initial probability density $p_0(X_0)$ and N simulated state values $(\tilde{X}_0^1, \ldots, \tilde{X}_0^N)) \sim p_0(X_0)$, then at time t:

(i) Sampling Step: $(\tilde{X}_t^1, \ldots, \tilde{X}_t^N) \sim p_t^N$ where p_t^N is the estimated state conditional density.

(ii) Evolving Step: for $i = 1..N$, $(\tilde{X}_t^i) \longrightarrow (\tilde{X}_{t+1}^i, \tilde{Y}_{t+1}^i)$ by simulation of model (1)-(2).

(iii) Estimation Step: $p_{t+1}^N(X_{t+1}|Y_{1:t+1}) = \dfrac{\sum_{i=1}^N K_{2,\delta_N}(Y_{t+1} - \tilde{Y}_{t+1}^i) K_{1,\delta_N}(X_{t+1} - \tilde{X}_{t+1}^i)}{\sum_{i=1}^N K_{2,\delta_N}(Y_{t+1} - \tilde{Y}_{t+1}^i)}$

with $K_{1,\delta_N}(x) = \delta_N^{-s} K_1\left(\frac{x}{\delta_N}\right)$, $x \in \mathbb{R}^s$ and $K_{\delta_N}(y) = \delta_N^{-q} K_2\left(\frac{y}{\delta_N}\right)$, $y \in \mathbb{R}^q$.

This algorithm provides an "on line" L_1-convergent estimate of the density $p_t(X_t|Y_{1:t})$ when the particles number N tends to infinity:

Theorem 2 (a.s. L_1-convergence) *If K_1 and K_2 are positive bounded Parzen-Rosenblatt kernels, if δ_N is decreasing with N, if $p(\cdot|Y_{1:t-1})$ is positive and continuous at y_t and if there exist $M > 0$ such that $p(Y_t|X_t) \leq M$ for all t and $\alpha \in]-1, 0[$ such that $\delta_N^{2q} = O(N^\alpha)$, then*

$$\lim_{N\to\infty} \frac{N\delta_N^{s+q}}{\log N} = \infty \Longrightarrow \lim_{N\to\infty} \int |p_t^N(X_t|Y_{1:t}) - p_t(X_t|Y_{1:t})| \mathrm{d}x_t = 0 \quad a.s.$$

Proof: This theorem is proved in ((Rossi, 2004) and (Rossi & Vila, 2004)).

The R-CF and more generally the particle filters are powerful tools to deal with hidden Markov processes. But from a practical point of view it is more relevant to consider uncertain

hidden Markov processes. More precisely an unknown fixed parameter θ is supposed to be present in the model equation (1) or (2).

The natural approach, according to the particle filter principle, consists in setting a prior probability law $p_0(\theta)$ on the parameter θ and considering a new state, $Z_t = (X_t, \theta_t)$, which gathers all the unknown quantities. As θ is fixed the natural dynamic is $\theta_{t+1} = \theta_t$. Then the posterior law $p(Z_t|Y_{1:t})$ is approximated using particle filters and the previous algorithm immediately adapts to this context.

However, the natural dynamic $\theta_{t+1} = \theta_t$ for the parameter, although theoretically well adapted, leads to the divergence of the standard particle filters. This inefficiency results from the fact that the parameter space is only explored at the initialization step of the classic particle filter algorithms, which causes the impoverishment of the variety of the relevant particles. Fortunately the R-CF is not affected by this drawback. Indeed, its smooth approach ensures a good parameter space exploration throughout the filtering procedure. In addition, results of consistency, for the R-CF with unknown parameter, has been established in ((Rossi, 2004) and (Rossi & Vila, 2005)).

Application

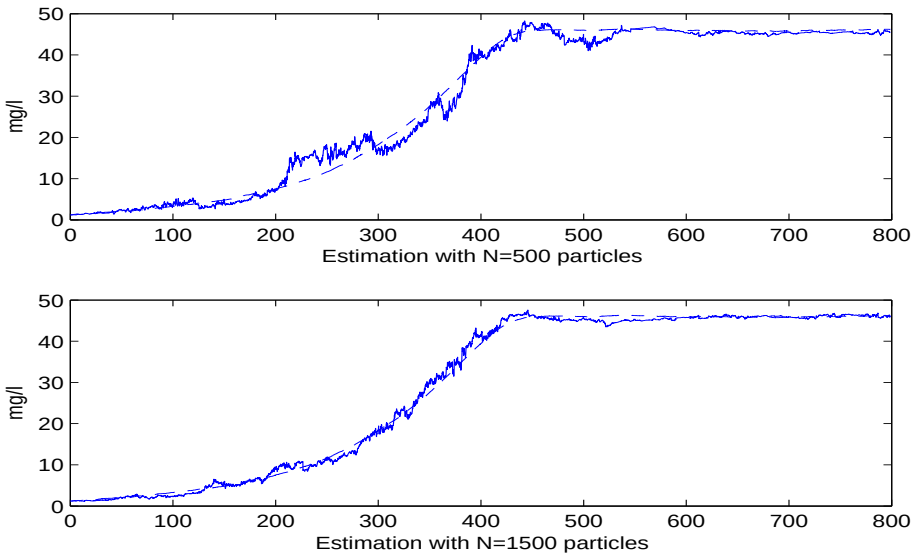

Figure 3: Biomass concentration estimation with the R-CF (true values - -, estimations –)

Let us consider the biological process introduced in section 2.3.with the parameter setting of section 3.1. Here the problem is to estimate the biomass concentration using only the measure of the substrate concentration given by equation (6). In addition we also assumed that the constant μ_{max} is unknown. The prior law used for μ_{max} was $p_0(\mu_{max}) = \mathcal{U}[0\ 0.2]$, the uniform distribution over [0 0.2]. Figure 3 displays the R-CF estimation of the biomass concentration, over 800 hours.

To make easy the comparison between the true value X_t and the estimation, we took a punctual state estimate instead of the state posterior density. The estimate used is $\hat{X}_t^N = mean(\tilde{X}_t^1, \ldots, \tilde{X}_t^N)$, with $(\tilde{X}_t^1, \ldots, \tilde{X}_t^N)$ obtained in step (i) of the R-CF algorithm. Under suitable assumptions $\hat{X}_t^N$ is a consistent estimate of $E[X_t|Y_{1:t}]$ ((Rossi, 2004)). As shown by Figure 3, despite of uncertainties on the model, the R-CF filter provided good estimates of the biomass concentrations. The theoretical properties of convergence as $N \to \infty$ are well illustrated as can be shown: the estimation obtained with $N = 1500$ particles is more accurate than the one with $N = 500$.

4. NONPARAMETRIC ADAPTIVE AND PREDICTIVE CONTROL

The objective considered in this section is to find a control sequence $(U_t)_{t\geq 1}$ which forces the state variables $(X_t)_{t\geq 1}$, to follow as best as possible a given bounded trajectory $(X_t^*)_{t\geq 1}$. The state variable X_t is now again supposed to be observed and to evolve according to model (3), with function f_t completely or partly unknown.

Two control strategies are considered in the following, according to the immediate or anticipative trajectory fitness considered.

4.1. ADAPTIVE TRACKING CONTROL

Consider the particular case of model (4) suitable for the biotechnological systems, in which g_t is unknown. An adaptive control policy has to be built from the nonparametric estimate $\widehat{g}_t$ (8), which ensures the stochastic closed-loop stability. This last property is indeed necessary to ensure the convergence properties of the kernel estimator $\widehat{g}_t$.

An a priori knowledge about function g_t is then required; we assume that there exists a continuous function g^* and two constants $c_g \in [o, 1/2)$ and $C_g \in (0, \infty)$ such that, for all

$x \in \mathbb{R}^s, t \in \mathbb{N}$, $\|g_t(x) - g^*(x)\| \leq c_g \|x\| + C_g$. Function g^* characterizes the a priori knowledge about functions g_t and allows to compensate the possible lack of observations which could disrupt the local estimator $\widehat{g}_t$. When B_t is supposed to be invertible with respect to U_t, the adaptive control law is the solution U_t such that

$$B_t(X_t, U_t) = X^*_{t+1} - A_t(X_t)\widehat{g}_t(X_t)\mathbb{1}_{E_t}(X_t) - A_t(X_t)g^*(X_t)\mathbb{1}_{E_t^c}(X_t) \tag{13}$$

where $E_t := \{X_t \in \{x : \|\widehat{g}_t(x) - g^*(x)\| \leq d_g \|x\| + D_g\}\}$, $d_g \in (c_g, 1-c_g)$ and $D_g \in (C_g, \infty)$. E_t^c denotes the complementary set of E_t. The set E_t is introduced to ensure the closed-loop stability of model (4).

The control law (13) satisfies the following properties:

1. Stability of the closed loop : the following sufficient condition is satisfied
$$\limsup_{t\to\infty} \frac{1}{t+1} \sum_{i=0}^{t} \|X_i\|^{sup(2, \frac{m_\varepsilon}{2})} \leq Cte < \infty \qquad a.s.$$
2. Almost sure uniform convergence of $\widehat{g}_t$ to g on dilated compacts
3. Asymptotic optimality: $\frac{1}{t}\sum_{i=1}^{t} \|X_i - X_i^*\|^2 \xrightarrow{a.s.} trace(\Gamma)$ as $t \to \infty$, where Γ denotes the covariance matrix of the noise ε_t.

See (Portier & Oulidi, 2000) and (Hilgert, 1997) for more details.

Application

Figure 4 shows a simulation result obtained with the adaptive controller (13) used to regulate the substrate concentration around the reference value $S^* = 15\ mg.l^{-1}$. The a priori knowledge on model (6) was given by a Tessier model for the growth rate $\mu(s)$, with a priori values of the parameters μ_{max} and θ different from the ones used in the simulations. Constants d_f and D_f were conservatively chosen: $d_f = 1/2$ and $D_f = 1$. The adaptive controller revealed good tracking properties. A real experiment to control an anaerobic fluidized bed reactor was also presented in (Hilgert, Harmand, Steyer, & Vila, 2000).

4.2. OPTIMAL PREDICTIVE CONTROL

Let us consider again state model (3) with unknown function f_t and still the assumption of observed X_t.

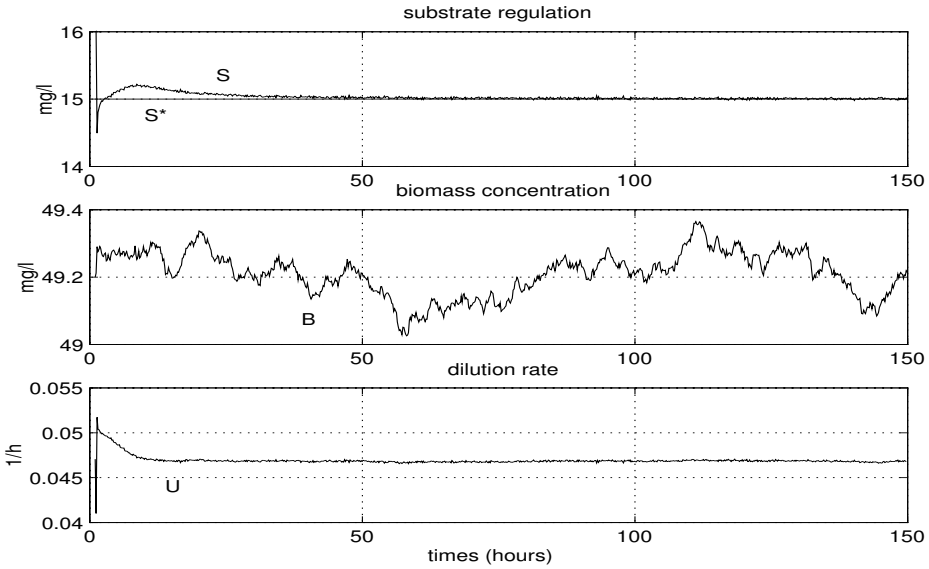

Figure 4: Simulation of the bioprocess(6) controlled with the nonparametric adaptive control law.

The principle of the so-called predictive control is now well-known among control theorists (see for example (Camacho & Bordons, 1995)). The specificity of predictive control is to consider both the future values of the state system and that of the reference trajectory, in a near forward horizon of given length H. More precisely at each time step the future values of the state variables on the horizon are predicted conditionally to intermediary control values. These control values are then optimized in order to minimize some discrepancy function between the predicted state values and that of the reference trajectory on the same horizon. The first of these optimal values of the control variable is then applied to the system which enters then the following time step and the predictive horizon is translated. Such an anticipating policy confers to predictive control a significant advantage among on-line tracking control policies, and is particularly adapted to the control of processes with slow dynamic such as the biotechnological processes. The main question raised by the predictive control algorithms is that of the stability of the closed loop. For deterministic systems several constraint conditions have been designed to ensure this stability (see (Mayne, Rawlings, Rao, & Scokaert, 2000) for a recent survey). For stochastic system this issue is still open for the general case. We consider it in the nonparametric approach to follow and solve it in a simple

case.

NPPC : a nonparametric predictive control algorithm for uncertain system:

At step t,

- let $J_t = \sum_{j=1}^{j=H} \| X^*_{t+j} - f^j_{t+j-1}\left(u^1, \ldots, u^j \,|\, X_{i,\, i\leq t}\,;\, U_{i,\, i\leq t-1}\right) \|^2$

  where

  - H is the chosen length of the receding horizon
  - $\widehat{X}_{t+j} = f^j_{t+j-1}\left(u^1, \ldots, u^j \mid X_{i,\, i\leq t}\ ;\ U_{i,\, i\leq t-1}\right)$ is a consistent estimate to be looked for $\mathrm{E}\left[X_{t+j} \,|\, X_{i,\, i\leq t}\ ;\ U_{i,\, i\leq t-1}\ ;\ U_t = u^1, \ldots, U_{t+j-1} = u^j\right]$ which is itself the minimum variance predictor of the state value X_{t+j}.

- Find $\bar{U}_t = (U^1_t, \ldots, U^H_t) = \mathrm{argmin}_{\|u^1\|\leq M, \ldots, \|u^H\|\leq M}\ J_t$

  with M: upper bound constraint in the control values.

- take $U_t = U^1_t$ and $t = t+1$

A j-step-ahead nonparametric state predictor:

Let $Z^j_t = (X_t, U_t, \ldots, U_{t+j-1})^t$. Let us consider as estimate of $\mathrm{E}(X_{t+j} \mid Z^j_t = z)$:

$$\widehat{X}_{t+j} = \widehat{\mathrm{E}}(X_{t+j} \mid Z^j_t = z) = \frac{\sum_{i=1}^{t-j} \delta_i^{-(s+jm)} K\left(\frac{z - Z^j_i}{\delta_i}\right) X_{i+j}}{\sum_{i=1}^{t-j} \delta_i^{-(s+jm)} K\left(\frac{z - Z^j_i}{\delta_i}\right)} \tag{14}$$

where K is a kernel of dimension $(s + jm)$.

For uncontrolled processes, the asymptotic behaviour of $\widehat{X}_{t+j}$ has been characterized under mixing conditions and stationarity assumptions (Bosq, 1996). These results are not easily applicable to the controlled processes we consider in this paper since the applied control values are state dependent.

However for the simplest case, $H = 1$, the following important results have been established (Wagner, 2001; Wagner & Vila, 2001):

- Let $j = 1$ in (14). Then $\widehat{X}_{t+1} \equiv \widehat{f}_t$, as given by (7).

- Let $\{\zeta_t\}$ be an observed m-dimensional Gaussian white noise independent of $\{\varepsilon_t\}$.

- Let $\{\gamma_t\}$ be a real positive series decreasing to 0 such that

  $\gamma_t = C(\log\log t)^{-\theta}$, in which C is a strictly positive constant and $\theta \in]0, 1/2[$.

- Let E_t be a particular subset of the state space, defined from the kernel estimate $\widehat{f}_t$ and from f^* a prior estimate of f_t (see (Wagner, 2001)) and generalizing the similar subset defined previously in 4.1.

- Let us consider the following one-step-ahead predictive control law:

  - If $X_t \in E_t : U_t = \widehat{U}_t = \text{argmin}_{||u||<M} \|X^*_{t+1} - \widehat{f}_t(X_t, u)\|^2 + \gamma_t\zeta_t$ (15)

  - If $X_t \in E_t^c : U_t = U_t^* = \text{argmin}_{||u||<M} \|X^*_{t+1} - f^*_t(X_t, u)\|^2 + \gamma_t\zeta_t$ (16)

Results : the control law (15)-(16) satisfies the following properties (Wagner, 2001):

1. Stability of the closed loop : the following sufficient condition is satisfied

$$\limsup_{t\to\infty} \frac{1}{t+1} \sum_{i=0}^{t} \|X_i\|^{sup(2, \frac{m_\varepsilon}{2})} \leq Cte < \infty \qquad a.s.$$

2. Almost sure uniform convergence of $\widehat{f}_t$ to f on dilated compacts

$$\sup_{||x,u|| \leq v_t} \|\widehat{f}_t(x, u) - f(x, u)\| = o(1) \qquad a.s.$$

   for $\{v_t\}$ such that $v_t = O\big(\gamma_t(\log\log t)^{\frac{1}{2}}\big)$.

3. Sub-optimality of the control

- for $\{f_t\}$ such that $f_t \overset{t\to\infty}{\to} f$ uniformly almost surely

$$\begin{aligned}\limsup_{n\to\infty}\frac{1}{n}\sum_{i=1}^{n}\parallel X_{i+1}-X^*_{i+1}\parallel^2 \;&\leq\; \mathrm{Tr}(\Gamma)\\ &+\; \limsup_{n\to\infty}\frac{1}{n}\sum_{i=1}^{n}\parallel \widehat{f}_i(X_i,\widehat{U}_i)-X^*_{i+1}\parallel^2 \mathbb{1}_{E_i}(X_i)\\ &+\; \limsup_{n\to\infty}\frac{1}{n}\sum_{i=1}^{n}\parallel f^*(X_i,U^*_i)-X^*_{i+1}\parallel^2 \mathbb{1}_{E_i^c}(X_i)\end{aligned}$$

- for *i.i.d.* $\{f_t\}$ such that $\mathrm{E}[f_t]=f$

$$\begin{aligned}\limsup_{n\to\infty}\frac{1}{n}\sum_{i=1}^{n}\parallel X_{i+1}-X^*_{i+1}\parallel^2 \;&\leq\; \mathrm{Tr}(\Gamma)\\ &+\; \big[\limsup_{n\to\infty}\{\frac{1}{n}\sum_{i=1}^{n}\parallel \widehat{f}_i(X_i,\widehat{U}_i)-X^*_{i+1}\parallel^2 \mathbb{1}_{E_i}(X_i)\}^{\frac{1}{2}}+(\mathrm{E}(\parallel f_1-f\parallel^2_\infty))^{\frac{1}{2}}\big]^2\\ &+\; \limsup_{n\to\infty}\frac{1}{n}\sum_{i=1}^{n}\parallel f^*(X_i,U^*_i)-X^*_{i+1}\parallel^2 \mathbb{1}_{E_i^c}(X_i).\end{aligned}$$

As it could have been expected, the effect of the stationary stochastic variability of $\{f_t\}$ in the case of *i.i.d.* $\{f_t\}$ increases the upper bound of the asymptotic mean squared error, with respect to the case of converging $\{f_t\}$.

Remark 1: the introduction of the prior information f^* about the unknown f and that of the decreasing stochastic excitation $\gamma_t\zeta_t$ into the one-step-ahead predictive control law (15)-(16), have only a theoretical interest to prove the stability of the closed loop and then the almost sure uniform convergence of $\widehat{f}_f$ to f. All the real case studies we performed in different control situations showed that these factors have no practical importance and can be ignored in practice by taking simply for all t

$$U_t \;=\; \widehat{U}_t \;=\; \mathrm{argmin}_{||u||<M}\|X^*_{t+1}-\widehat{f}_t(X_t,u)\|^2. \tag{17}$$

Remark 2: again, for $H\geq 2$, the general algorithm NPPC coupled with the nonparametric state predictor (14) performed satisfactorily on all the case studies considered, even if its theoretical statistical properties have not been characterized.

Remark 3: the minimization of the criterion function J_t at step t of the algorithm NPPC with respect to the constrained control variables $(u^1, \cdots, u^H)$, can be done by standard descent algorithm. We developed also a more efficient neural network-based minimization procedure and applied it online on a real biotechnological depollution process (Vila & Wagner, 2003).

Remark 4: the choice of the length of the predictive horizon H must result from a case by case compromise between long term optimality of the predictive control (high values for H) and the quality of the kernel predictors (low values).

Application

Let us considered again the case of the generic single biomass / single substrate biotechnological process given by (6), which enters model (3) with

$$X_t = \begin{pmatrix} B_t \\ S_t \end{pmatrix}, U_t\text{: dilution rate, } f_t(X_t, U_t) = \begin{pmatrix} (1 + T(\mu_t - U_t))X_t^{(1)} \\ X_t^{(2)} - T\mu_t X_t^{(1)}/\tau + U_t(S_{in} - X_t^{(2)})T \end{pmatrix}.$$

The process was simulated with the same setting as in section 3.1. . The simulation was carried on $L = 900$ time steps of $1h$, starting from $B_0 = 40mg.l^{-1}$, $S_0 = 20mg.l^{-1}$.

The reference substrate trajectory S^* to be followed by the process was given by a step function varying between 22 and $30mg/l$.

In order to test the performance of the NPPC algorithm several predictive control strategies have been computed for different values of the predictive horizon H, *under the hypothesis that model function $f_t(X_t, U_t)$ was completely unknown with state predictions computed according to (14).*

Results: the residual sum of squares (RSS) between the reference and controlled trajectories have been computed for 10 simulation runs for $H = 1, \ldots, 10$:

Horizon H	1	2	3	4	5	6	7	8	9	10
RSS	283.23	253.29	248.21	234.26	236.93	232.47	223.89	220.68	233.76	236.19

In order to assess the efficiency of the NPPC algorithm, the same control processing simulations were carried on under the hypothesis of a full knowledge of the process model f_t. In

that case, the j-step-ahead predictor has been computed by the deterministic recurrence

$$\widehat{X}_{t+j} = f_{t+j-1}(\widehat{X}_{t+j-1}, U_{t+j-1}) \tag{18}$$

leading to the following results:

Horizon H	1	2	3	4	5	6	7	8	9	10
RSS	276.74	264.12	219.12	216.56	192.26	159.53	148.98	183.86	120.86	214.91

Comments

- As expected, when H increases the tendency of the RSS is to decrease. However the lower efficiency of the nonparametric predictor (14) and that of the deterministic recurrence predictor (18) as j increases, disturb this trend.

- A comparison of the two arrays reveals that the nonparametric prediction and control approach is able to retrieve most of the information supplied by the full knowledge of the model function f_t.

5. CONCLUSION AND PERSPECTIVES: TOWARDS THE NONPARAMETRIC SUPERVISION OF UNCERTAIN SYSTEMS

When dealing with process control, an unavoidable issue is that of supervision. Supervision involves being able to detect any default in the process (*e.g.* pump clogging in a bioprocess), locating the default and remedying it (by an appropriate sequence of actions). From a statistical point of view, the problems of detection and isolation of a default are equivalent to detecting abrupt changes in a stochastic process, and testing multiple hypotheses to determine the faulty scenario among a number of possible scenarii of defaults (Dubuisson, 2001).

There exist many statistical procedures to answer such questions, see (Basseville & Nikiforov, 1993). A well-known one is the CuSum test. It is based on a comparison, at each time instant, of the difference between the log-likelihood ratio value and its current minimal value, with respect to a fixed threshold. Most of these techniques require knowledge of both state and observation models.

When the state model is uncertain, the question is still open. However combining nonparametric estimates as (7) or (8) with classical test procedures gave us encouraging results on real experimental data issued from a depollution process (Hilgert, Verdier, & Vila, 2006).

Moreover, introducing filtering methods such as the one proposed above, will allow to generalize these nonparametric detection procedures to the most frequent situation of indirectly observed systems described by models (1) and (2).

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